

Schwinger Mechanism for Gluon Pair Production in the Presence of Arbitrary Time Dependent Chromo-Electric Field in Arbitrary Gauge

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Abstract

We study non-perturbative gluon pair production from arbitrary time dependent chromo-electric field $E^a(t)$ with arbitrary color index $a = 1, 2, \dots, 8$ via Schwinger mechanism in arbitrary covariant background gauge α . We show that the probability of non-perturbative gluon pair production per unit time per unit volume per unit transverse momentum $\frac{dW}{d^4x d^2p_T}$ is independent of gauge fixing parameter α . Hence the result obtained in the Fynman-'t Hooft gauge, $\alpha=1$, is the correct gauge invariant and gauge parameter α independent result.

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An exact non-perturbative result for electron-positron pair production from a constant electric field was obtained by Schwinger in 1951 by using proper time method [1]. In QCD this result depends on two independent casimir/gauge invariants $C_1 = [E^a E^a]$ and $C_2 = [d_{abc} E^a E^b E^c]^2$ with color indices $a, b, c=1,2,\dots,8$ in SU(3) [2, 3]. Recently, using shift theorem [4], we have extended this study to arbitrary time dependent electric field $E(t)$ in QED [5] and to arbitrary time dependent chromo-electric field $E^a(t)$ in QCD [6, 7]. This result crucially depends on the validity of the shift conjecture which is not yet established.

In [7] Schwinger mechanism for gluon pair production from arbitrary time dependent chromo-electric field $E^a(t)$ was studied in the Feynman-t'hooft gauge $\alpha=1$. In this paper we will extend this study to any arbitrary gauge fixing parameter α . We find that the result is gauge fixing parameter α independent. Hence the result obtained in [7] in Feynman-'t Hooft gauge, $\alpha=1$, is the correct gauge invariant and gauge parameter α independent result.

The following result was obtained for the probability of gluon pair production from arbitrary time dependent chromo-electric field $E^a(t)$ in $\alpha=1$ gauge via Schwinger mechanism [7]:

$$\frac{dW_{g(\bar{g})}}{dt d^3x d^2p_T} = \frac{1}{4\pi^3} \sum_{j=1}^3 |g\Lambda_j(t)| \ln[1 + e^{-\frac{\pi p_T^2}{|g\Lambda_j(t)|}}]. \quad (1)$$

In the above equation

$$\Lambda_1^2 = \frac{C_1(t)}{2} [1 - \cos\theta(t)]; \quad \Lambda_{2,3}^2 = \frac{C_1(t)}{2} [1 + \cos(\frac{\pi}{3} \pm \theta(t))]; \quad \cos 3\theta(t) = -1 + 6C_2(t)/C_1^3(t) \quad (2)$$

$$\text{where} \quad C_1(t) = [E^a(t)E^a(t)]; \quad C_2(t) = [d_{abc}E^a(t)E^b(t)E^c(t)]^2 \quad (3)$$

are two independent time-dependent casimir/gauge invariants in SU(3).

We will present a proof of gauge fixing parameter α independence of eq. (1) in the following.

In the background field method of QCD [8, 9] the gauge field is the sum of classical chromo-field A_μ^a and the quantum gluon field Q_μ^a . The non-abelian field tensor becomes

$$F_{\mu\nu}^a[A + Q] = \partial_\mu(A_\nu^a + Q_\nu^a) - \partial_\nu(A_\mu^a + Q_\mu^a) + gf^{abc}(A_\mu^b + Q_\mu^b)(A_\nu^c + Q_\nu^c). \quad (4)$$

The gauge field Lagrangian density is

$$\mathcal{L}_{gl} = -\frac{1}{4}F_{\mu\nu}^a[A + Q]F^{\mu\nu a}[A + Q] - \frac{1}{2\alpha}[D_\mu[A]Q^{\mu a}]^2 \quad (5)$$

where the second term in the right hand side is the gauge fixing term which depends on the background field A_μ^a [8, 9]. The covariant derivative is given by

$$D_\mu^{ab}[A] = \delta^{ab}\partial_\mu + gf^{abc}A_\mu^c. \quad (6)$$

Keeping terms up to quadratic in Q field (for gluon pair production) we find from eq. (5)

$$\begin{aligned} \int d^4x \mathcal{L} &= \frac{1}{2} \int d^4x [-(D_\mu[A]Q_\nu^a)F^{\mu\nu a}[A] + Q^{\mu a}M_{\mu\nu}^{ab}[A]Q^{\nu b}] \\ &= \frac{1}{2} \int d^4x [(D_\mu[A]F^{\mu\nu a}[A])Q_\nu^a + Q^{\mu a}M_{\mu\nu}^{ab}[A]Q^{\nu b}] \end{aligned} \quad (7)$$

where

$$M_{\mu\nu}^{ab}[A] = g_{\mu\nu}[D_\rho(A)D^\rho(A)]^{ab} - 2gf^{abc}F_{\mu\nu}^c[A] + \left(\frac{1}{\alpha} - 1\right)(D_\mu[A]D_\nu[A])^{ab} \quad (8)$$

with $g_{\mu\nu} = (1, -1, -1, -1)$.

The vacuum-to-vacuum transition amplitude for gluon is given by

$$\langle 0|0 \rangle^A = \frac{Z[A]}{Z[0]} = \frac{\int [dQ] e^{i \int d^4x [Q^{\mu a}M_{\mu\nu}^{ab}[A]Q^{\nu b} + (D_\mu[A]F^{\mu\nu a}[A])Q_\nu^a]}}{\int [dQ] e^{i \int d^4x Q^{\mu a}M_{\mu\nu}^{ab}[0]Q^{\nu b}}}. \quad (9)$$

To evaluate the path integration in eq. (9) we change the variable

$$Q_\mu^a(x) = Q'_\mu{}^a(x) - \frac{1}{2} \int d^4x' G_{\mu\nu}^{ab}(x, x') D_\lambda(x') F^{\lambda\nu b}(x') \quad (10)$$

where we denote $D_\mu^{ab}(x) = D_\mu^{ab}[A](x)$ and $F_{\mu\nu}^a(x) = F_{\mu\nu}^a[A](x)$. The Green's function is given by (using Schwinger's notation [1])

$$G_{\mu\nu}^{ab}(x, x') = [\langle x | \frac{1}{M} | x' \rangle]_{\mu\nu}^{ab} = [\langle x | \int_0^\infty ds e^{-sM} | x' \rangle]_{\mu\nu}^{ab}. \quad (11)$$

Under this change of variable we find from eq. (9)

$$\langle 0|0 \rangle^A = \frac{Z[A]}{Z[0]} = e^{-iS_{\text{tad}}} \times e^{iS^{(1)}} \quad (12)$$

where

$$S_{\text{tad}} = \frac{1}{2} \int d^4x \int d^4x' D_\mu(x) F^{\mu\lambda a}(x) G_\lambda^{\nu ab}(x, x') D^\sigma(x') F_{\sigma\nu b}(x') \quad (13)$$

is the tadpole (or single gluon) effective action and

$$S^{(1)} = -i \ln \left[\frac{\text{Det}^{-1/2} M_{\mu\nu}^{ab}[A]}{\text{Det}^{-1/2} M_{\mu\nu}^{ab}[0]} \right] = \frac{i}{2} \text{Tr} [\ln M_\mu^{\nu, ab}[A] - \ln M_\mu^{\nu, ab}[0]] \quad (14)$$

is the one loop (or gluon pair) effective action.

The trace Tr is given by

$$\text{Tr}\mathcal{O} = \text{tr}_{\text{Lorentz}}\text{tr}_{\text{color}} \int d^4x \langle x|\mathcal{O}|x \rangle. \quad (15)$$

We choose the arbitrary time-dependent chromo-electric field $E^a(t)$ to be along the z -axis (the beam direction) and work in the choice $A_3^a = 0$ so that

$$A_\mu^a(x) = -\delta_{\mu 0} E^a(t) z \quad (16)$$

is non-vanishing. The color indices are arbitrary, $a=1,2,\dots,8$.

It can be seen that the tadpole effective action S_{tad} in eq. (13) depends on the gauge fixing parameter α via the Green's function $G_\lambda^{\nu ab}(x, x')$. However, we will show in the appendix that the tadpole effective action S_{tad} per unit time per unit volume and per unit transverse momentum ($\frac{dS_{\text{tad}}}{d^4x d^2p_T}$) is zero for any non-vanishing transverse momentum. Hence there is no tadpole contribution to eq. (1) and we will not consider it any more.

We write eq. (8) as

$$M_{\mu\nu}^{ab}[A] = M_{\mu\nu, \alpha=1}^{ab}[A] + \alpha' (D_\mu[A] D_\nu[A])^{ab} \quad (17)$$

where

$$\alpha' = \left(\frac{1}{\alpha} - 1\right) \quad (18)$$

and

$$M_{\mu\nu, \alpha=1}^{ab}[A] = g_{\mu\nu} [D_\rho(A) D^\rho(A)]^{ab} - 2g f^{abc} F_{\mu\nu}^c[A]. \quad (19)$$

Hence we find

$$\begin{aligned} \text{Trln} M_\mu^{\nu, ab}[A] &= \text{Trln} [M_{\mu, \alpha=1}^\lambda[A] \{ \delta_\lambda^\nu + \alpha' M_{\lambda, \alpha=1}^{-1 \sigma}[A] (D_\sigma[A] D^\nu[A]) \}]^{ab} \\ &= \text{Trln} [M_{\mu, \alpha=1}^{\nu, ab}[A]] + \text{Trln} [\delta_\mu^\nu \delta^{ab} + \alpha' [M_{\mu, \alpha=1}^{-1 \lambda}[A] D_\lambda[A] D^\nu[A]]^{ab}]. \end{aligned} \quad (20)$$

Since $\text{Trln} [M_{\mu, \alpha=1}^{\nu, ab}[A]]$ was studied in [7] we will evaluate the $\alpha' = (\frac{1}{\alpha} - 1)$ dependent term in this paper.

The ghost determinant is evaluated in [7] where we have used $\alpha=1$. Since the ghost Lagrangian density is α independent [7], we do not discuss ghost in this paper. Whenever we mention $\alpha = 1$ case in this paper, we assume that the ghost contribution is included.

Using eq. (15) we find

$$\begin{aligned} \text{Tr} \ln [\delta_\mu^\nu \delta^{ab} + \alpha' [M_{\mu, \alpha=1}^{-1 \lambda} [A] D_\lambda [A] D^\nu [A]]^{ab}] &= \text{tr}_{\text{Lorentz}} \text{tr}_{\text{color}} [\int d^2 x_T < x_T | \int_{-\infty}^{+\infty} dt < t | \\ \int_{-\infty}^{+\infty} dz < z | \ln [\delta_\mu^\nu + \alpha' M_{\mu, \alpha=1}^{-1 \lambda} D_\lambda D^\nu] | z > | t > | x_T >]^{ab}. \end{aligned} \quad (21)$$

Using eq. (16) in (6) we find

$$D_\mu^{ab} [A] = \partial_\mu \delta^{ab} - \delta_{\mu 0} z i g \Lambda^{ab}(t) \quad (22)$$

where

$$\Lambda^{ab}(t) = i f^{abc} E^c(t). \quad (23)$$

Using the relation

$$[D_\mu [A], D_\nu [A]]^{ab} = -g f^{abc} F_{\mu\nu}^c \quad (24)$$

and by using shift theorem [4] (by shifting $[z \rightarrow z + \frac{i}{g\Lambda(t)} \frac{d}{dt}]^{ab}$) we find from eq. (21)

$$\begin{aligned} \text{Tr} \ln [\delta_\mu^\nu \delta^{ab} + \alpha' [M_{\mu, \alpha=1}^{-1 \lambda} [A] D_\lambda [A] D^\nu [A]]^{ab}] &= \text{tr}_{\text{Lorentz}} \text{tr}_{\text{color}} [\int d^2 x_T < x_T | \int_{-\infty}^{+\infty} dt < t | \\ \int_{-\infty}^{+\infty} dz < z + \frac{i}{g\Lambda(t)} \frac{d}{dt} | \ln [\delta_\mu^\nu + \alpha' (M')_{\mu, \alpha=1}^{-1 \lambda} D'_\lambda D'^\nu] | z + \frac{i}{g\Lambda(t)} \frac{d}{dt} > | t > | x_T >]^{ab} \\ &= \text{tr}_{\text{Lorentz}} \text{tr}_{\text{color}} [\int d^2 x_T < x_T | \int_{-\infty}^{+\infty} dt < t | \\ \int_{-\infty}^{+\infty} dz < z + \frac{i}{g\Lambda(t)} \frac{d}{dt} | \ln [\delta_\mu^\nu + \alpha' D'_\mu \frac{1}{(D')^2} D'^\nu] | z + \frac{i}{g\Lambda(t)} \frac{d}{dt} > | t > | x_T >]^{ab} \end{aligned} \quad (25)$$

where

$$D'_\mu^{ab} [A] = (1 - \delta_{\mu 0}) \delta^{ab} \partial_\mu - \delta_{\mu 0} z i g \Lambda^{ab}(t) \quad (26)$$

(μ is not summed) and

$$M'^{ab}_{\mu\nu, \alpha=1} [A] = g_{\mu\nu} [D'_\rho(A) D'^\rho(A)]^{ab} - 2g f^{abc} F_{\mu\nu}^c [A]. \quad (27)$$

It has to be remembered that the z integration must be performed from $-\infty$ to $+\infty$ for the shift theorem [4] to be applicable.

Expanding the Logarithm in eq. (25) we find

$$\begin{aligned} \ln [\delta_\mu^\nu + \alpha' D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab} &= \alpha' [D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab} \\ &- \frac{\alpha'^2}{2} [D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab} + \frac{\alpha'^3}{3} [D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab} - \frac{\alpha'^4}{4} [D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab} \\ &+ \frac{\alpha'^5}{5} [D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab} - \dots \end{aligned} \quad (28)$$

Summing the series we obtain

$$\ln[\delta_\mu^\nu + \alpha' D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab} = \ln(1 + \alpha') [D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab}. \quad (29)$$

Using the cyclic properties of the trace ($\text{Tr}[D'_\mu \frac{1}{(D')^2} D'^\nu]^{ab} = \text{Tr}[D'^\nu D'_\mu \frac{1}{(D')^2}]^{ab}$) and using eq. (29) we find from eq. (25)

$$\begin{aligned} \text{Tr} \ln[\delta_\nu^\mu \delta^{ab} + \alpha' [M_{\mu, \alpha=1}^{-1 \lambda}[A] D_\lambda[A] D^\nu[A]]^{ab}] &= \text{tr}_{\text{Lorentz}} \text{tr}_{\text{color}} [\int d^2 x_T < x_T | \int_{-\infty}^{+\infty} dt < t | \\ &\int_{-\infty}^{+\infty} dz < z + \frac{i}{g\Lambda(t)} \frac{d}{dt} | D'_\mu \frac{1}{(D')^2} D'^\nu \ln[1 + \alpha'] | z + \frac{i}{g\Lambda(t)} \frac{d}{dt} > | t > | x_T >]^{ab} \\ &= \text{tr}_{\text{color}} [\int d^2 x_T < x_T | \int_{-\infty}^{+\infty} dt < t | \int_{-\infty}^{+\infty} dz < z + \frac{i}{g\Lambda(t)} \frac{d}{dt} | \ln[1 + \alpha'] | z + \frac{i}{g\Lambda(t)} \frac{d}{dt} > \\ &| t > | x_T >]^{ab} = 8 \ln[1 + \alpha'] \int d^4 x \int d^4 p = -8 \ln(\alpha) \int d^4 x \int d^4 p \end{aligned} \quad (30)$$

where we have used $\alpha' = (\frac{1}{\alpha} - 1)$ from eq. (18). Using eq. (30) in (20) we find

$$\text{Tr} \ln M_{\mu}^{\nu, ab}[A] = \text{Tr} \ln [M_{\mu, \alpha=1}^{\nu, ab}[A]] - 8 \ln(\alpha) \int d^4 x \int d^4 p. \quad (31)$$

Similarly for the free part we get

$$\text{Tr} \ln M_{\mu}^{\nu, ab}[0] = \text{Tr} \ln [M_{\mu, \alpha=1}^{\nu, ab}[0]] - 8 \ln(\alpha) \int d^4 x \int d^4 p. \quad (32)$$

Using eqs. (31) and (32) in eq. (14) we find

$$S^{(1)} = \frac{i}{2} \text{Tr} [\ln M_{\mu}^{\nu, ab}[A] - \ln M_{\mu}^{\nu, ab}[0]] = \frac{i}{2} \text{Tr} [\ln M_{\mu, \alpha=1}^{\nu, ab}[A] - \ln M_{\mu, \alpha=1}^{\nu, ab}[0]] \quad (33)$$

where the gauge parameter α dependence exactly cancelled from the interacting and free part. The imaginary part of this effective action $S^{(1)}$ gives real gluon pair production result (eq. (1)) [7]. Hence we find that the non-perturbative result for gluon pair production from arbitrary $E^a(t)$ via Schwinger mechanism is independent of arbitrary gauge parameter α which is used in the gauge fixing term in the background field method of QCD.

To conclude we have studied the Schwinger mechanism for gluon pair production in the presence of arbitrary time-dependent chromo-electric field $E^a(t)$ in arbitrary covariant background gauge α with arbitrary color index $a=1,2,..8$ by directly evaluating the path integral. We have found that the exact result for non-perturbative gluon pair production from arbitrary $E^a(t)$ via Schwinger mechanism is independent of arbitrary gauge parameter α which is used in the gauge fixing term in the background field method of QCD. We find

that the non-perturbative gluon pair production result from arbitray $E^a(t)$ via Schwinger mechanism is both gauge invariant and gauge parameter α independent. Gluon production from classical chromo field may be relevant to study production of quark-gluon plasma at RHIC and LHC.

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APPENDIX A:

By using eq. (16) in (6) we find

$$\begin{aligned} i(D^\mu[A])^{ab} &= \delta^{\mu 0}(\delta^{ab}\hat{p}_0 - g\Lambda^{ab}(t)z) + \delta^{ab}\delta^{\mu 1}\hat{p}_x + \delta^{ab}\delta^{\mu 2}\hat{p}_y + \delta^{ab}\delta^{\mu 3}\hat{p}_z \\ &= \delta^{\mu 0}(\delta^{ab}\hat{p}_0 - g\Lambda^{ab}(t)z) + \delta^{ab}\delta^{\mu T}\hat{p}_T + \delta^{ab}\delta^{\mu 3}\hat{p}_z \end{aligned} \quad (\text{A1})$$

where $\Lambda^{ab}(t)$ is given by eq. (23). From eqs. (8) and (11) we obtain

$$\begin{aligned} G_{\mu}^{\nu,ab}(x, x') &= \int_0^\infty ds [< x | \int_0^\infty ds \\ &e^{-s(\delta_{\mu}^{\nu}[-(\hat{p}_0 - g\Lambda(t)z)^2 + \hat{p}_z^2 + \hat{p}_T^2] - 2g\Lambda(t)\hat{F}_{\mu}^{\nu} - (\frac{1}{\alpha} - 1)(\delta_{\mu 0}(\hat{p}_0 - g\Lambda(t)z) - \delta_{\mu T}\hat{p}_T - \delta_{\mu 3}\hat{p}_z)(\delta^{\nu 0}(\hat{p}_0 - g\Lambda(t)z) + \delta^{\nu T}\hat{p}_T + \delta^{\nu 3}\hat{p}_z))} \\ &|x' >]^{ab}. \end{aligned} \quad (\text{A2})$$

where

$$\hat{F}_{\mu}^{\nu} = \delta_{\mu 3}\delta_{\nu 0} + \delta_{\mu 0}\delta_{\nu 3}. \quad (\text{A3})$$

Using eqs. (A2) and (16) in (13) we find the tadpole effective action

$$\begin{aligned} S_{\text{tad}} &= \frac{1}{2} \int d^2x_T d^2x'_T dz dt dz' dt' \int_0^\infty ds \frac{dE^a(t)}{dt} [< x_T | < t | < z | \\ &e^{-s(\delta_{\mu}^{\nu}[-(\hat{p}_0 - g\Lambda(t)z)^2 + \hat{p}_z^2 + \hat{p}_T^2] - 2g\Lambda(t)\hat{F}_{\mu}^{\nu} - (\frac{1}{\alpha} - 1)(\delta_{\mu 0}(\hat{p}_0 - g\Lambda(t)z) - \delta_{\mu T}\hat{p}_T - \delta_{\mu 3}\hat{p}_z)(\delta^{\nu 0}(\hat{p}_0 - g\Lambda(t)z) + \delta^{\nu T}\hat{p}_T + \delta^{\nu 3}\hat{p}_z))} \\ &|t' > |z' > x'_T >]^{ab} \frac{dE^b(t')}{dt'}. \end{aligned} \quad (\text{A4})$$

Inserting complete set of $|p_T >$ states (by using $\int d^2p_T |p_T > < p_T| = 1$) we find

$$\begin{aligned} S_{\text{tad}} &= \frac{1}{2} \int d^2x_T d^2x'_T dz dt dz' dt' d^2p_T \int_0^\infty ds \frac{dE^a(t)}{dt} [< x_T | p_T > < t | < z | \\ &e^{-s(\delta_{\mu}^{\nu}[-(\hat{p}_0 - g\Lambda(t)z)^2 + \hat{p}_z^2 + \hat{p}_T^2] - 2g\Lambda(t)\hat{F}_{\mu}^{\nu} - (\frac{1}{\alpha} - 1)(\delta_{\mu 0}(\hat{p}_0 - g\Lambda(t)z) - \delta_{\mu T}p_T - \delta_{\mu 3}\hat{p}_z)(\delta^{\nu 0}(\hat{p}_0 - g\Lambda(t)z) + \delta^{\nu T}p_T + \delta^{\nu 3}\hat{p}_z))} \\ &|t' > |z' > < p_T | x'_T >]_3^{3, ab} \frac{dE^b(t')}{dt'}. \end{aligned} \quad (\text{A5})$$

where $\overset{3}{3}$ means the $\mu = 3$ and $\nu = 3$ component of the Lorentz matrix. Using $\langle q|p \rangle = \frac{1}{\sqrt{2\pi}} e^{iqp}$ we obtain

$$S_{\text{tad}} = \frac{1}{2(2\pi)^2} \int d^2x_T d^2x'_T dz dt dz' dt' d^2p_T \int_0^\infty ds \frac{dE^a(t)}{dt} [e^{ix_T \cdot p_T} \langle t | \langle z | e^{-s(\delta_\mu^\nu [-(\hat{p}_0 - g\Lambda(t)z)^2 + \hat{p}_z^2 + p_T^2] - 2g\Lambda(t)\hat{F}_\mu^\nu - (\frac{1}{\alpha} - 1)(\delta_{\mu 0}(\hat{p}_0 - g\Lambda(t)z) - \delta_{\mu T}p_T - \delta_{\mu 3}\hat{p}_z)(\delta^{\nu 0}(\hat{p}_0 - g\Lambda(t)z) + \delta^{\nu T}p_T + \delta^{\nu 3}\hat{p}_z))} e^{-ix'_T \cdot p_T} |z' \rangle |t' \rangle]_3^{3ab} \frac{dE^b(t')}{dt'}. \quad (\text{A6})$$

Integrating over x'_T (by using $\int d^2x'_T e^{-ix'_T \cdot p_T} = (2\pi)^2 \delta^{(2)}(\vec{p}_T)$) we find

$$\frac{dS_{\text{tad}}}{dt d^3x d^2p_T} = \frac{\delta^{(2)}(\vec{p}_T)}{2} \int dt' \int dz' \int_0^\infty ds \frac{dE^a(t)}{dt} [\langle t | \langle z | e^{-s(\delta_\mu^\nu [(\hat{p}_0 - g\Lambda(t)z)^2 + \hat{p}_z^2 + p_T^2] - 2g\Lambda(t)\hat{F}_\mu^\nu - (\frac{1}{\alpha} - 1)(\delta_{\mu 0}(\hat{p}_0 - g\Lambda(t)z) - \delta_{\mu T}p_T - \delta_{\mu 3}\hat{p}_z)(\delta^{\nu 0}(\hat{p}_0 - g\Lambda(t)z) + \delta^{\nu T}p_T + \delta^{\nu 3}\hat{p}_z))} |z' \rangle |t' \rangle]_3^{3ab} \frac{dE^b(t')}{dt'}. \quad (\text{A7})$$

which has a $\delta^{(2)}(\vec{p}_T)$ distribution. Hence we find for any non-vanishing p_T

$$\frac{dS_{\text{tad}}}{d^4x d^2p_T} = 0. \quad (\text{A8})$$

Hence the tadpole (or single gluon) effective action do not contribute to eq. (1) of the non-perturbative gluon (pair) production rate $\frac{dW}{d^4x d^2p_T}$ via Schwinger mechanism.

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